Sequential Nested RI model: new take

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Introduction

This project: choice theory based on sequential info acquisition (RI)

What is RI: *Rational Inattention*

- Model of info acquisition!
- DM chooses information nonparametrically, controlling whole distribution of noise
- Mechanics: unknown state $\Rightarrow$ signal $\Rightarrow$ action
- Generate random choice data

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How does it differ from RUM?

Alternative take on randomness

Randomness

classical RUM

analyst has limited access to DM's preferences

$\Rightarrow$ unobserved part is random for him

$\Rightarrow$ for him choice is stochastic

if $v_i = u_i + \varepsilon_i$ with $\varepsilon_i \sim EV(0, \frac{1}{\lambda})$

then $P(i) = \frac{e^{\frac{u_i}{\lambda}}}{\sum_j e^{\frac{u_j}{\lambda}}}$

incomplete information (RI) model

analyst and DM do not know preferences

preferences are random

$\Rightarrow$ DM acquires info and learns her preferences

$\Rightarrow$ choice depends on info $\Rightarrow$ choice is random

for entropy cost of info $P(i|u) = \frac{e^{\frac{u_i}{\lambda} + \log P(i)}}{\sum_j e^{\frac{u_j}{\lambda} + \log P(j)}}$

Substitution patterns

MNL admits IIA $\Rightarrow$ very restrictive substitution pattern

Breaking IIA using RI:

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$$\mathbb{E}_U[U \cdot p(i|U)] - \text{Cost of Info}$$

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- This talk: RI breaks IIA **varying payoff structure**

Idea

Put additive structure on utilities

Inspiration from mixed logit:

$$\varepsilon = \varepsilon_{nest} + \varepsilon_{idio},$$

Sequential decision process:

1. DM may learn about common component
2. DM may learn about idiosyncratic component
3. DM chooses an option

Simplest model

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- Random option: $u = v + \eta$, both errors are binary independent r.v. with priors μ_v, μ_η

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- Parameters: DM chooses all three options

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learns about v

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learns about v

good info

bad info

Optimal behavior

Optimal info acquisition & choice procedure mimics nested logit

learns about v

good info
learns about η

bad info
chooses safe

Optimal behavior

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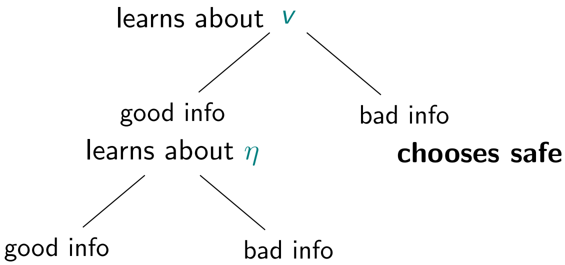

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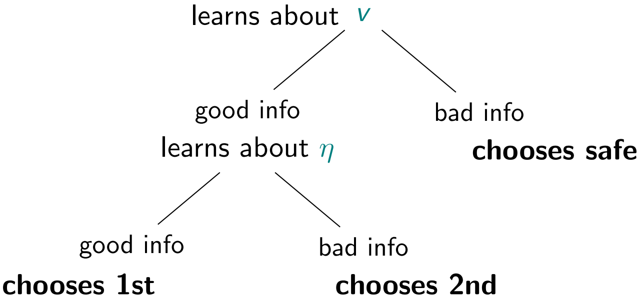

Stylized example

- Example: presearch + search

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- DM decides about vacation
  1. *Presearch*: check online average price level of tickets
  2. If **high**: stay home and save money, if **low**: book dates for vacation
  3. *Search*: **low** $\Rightarrow$ after a while choose exact airline among availible options

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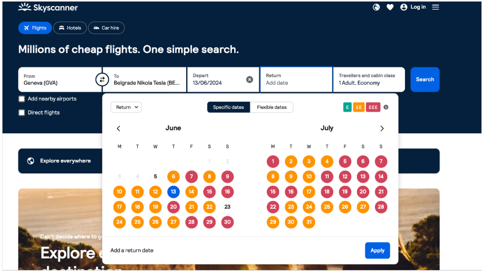

Figure: Presearch as yes/no decision

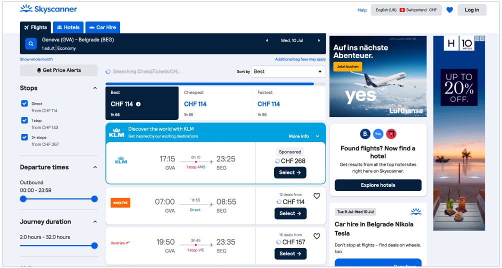

Figure: Search as choice of the best option

Solution

- Formula in (v_h, η_h) state:

$$P(1|v_h,\eta_h) = \frac{e^{\frac{v_h+\mathbb{E}V_2}{\lambda_1}+\log P(12)}}{e^{\frac{v_h+\mathbb{E}V_2}{\lambda_1}+\log P(12)} + e^{\frac{w}{\lambda_1}+\log P(3)}} \cdot \frac{e^{\frac{\eta_h}{\lambda_2}+\log P(1)}}{e^{\frac{\eta_h}{\lambda_2}+\log P(1)} + e^{\frac{\eta_l}{\lambda_2}+\log P(1)}},$$

  where $\mathbb{E}V_2$ is expected payoff from the risky nest

- Main departure from nested logit: dynamics + prior beliefs
  - Dynamic optimality: in first period DM takes into account optimal average payoff from second period
  - Prior beliefs: utility shifters $P(.)$

General substitution patterns

- What about IIA?

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- Fosgerau et al. (2020) critique of RI-logit:
  - Fix RI-logit setup
  - Add new state, in which only one payoff changes (price discount)
  - IIA for unchanged options between two states

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- Our case:
  - Composite state structure: (common, idio)
  - $\Rightarrow$ in new state *only* idio changes
  - $\Rightarrow$ IIA breaks thanks to "nested" procedure

Comparison with nested logit

Question: can simple nested logit recover substitution pattern from sequential nested RI logit?

Synthetic data generation:

1. Assume sequential nested RI logit
2. Solve the model numerically for set of parameters
3. Generate states and synthetic data
4. Estimate nested logit parameters: β ($\beta_{true} = 1$), λ

Answer: Usually nested logit performs poorly: over/underestimates correlation and β

. . . but not always!

Nested logit $\approx$ sequential nested RI logit

- Fix intermediate values of λ_1, λ_2, options are homogenous ex-ante

  $\Rightarrow$ in nested logit $\beta \approx 1$, $\lambda > 1$ and significant

  $\Rightarrow$ nested logit predicts average behavior very well

- Why? Symmetric mistakes for risky options mirrors nested logit substitution pattern

Overall

1. *Microfoundation*
   - Pros: "Nested" procedure as optimal sequential learning strategy
   - Cons: payoff structure is very ad-hoc
2. *Substitution*
   - Pros: richer substitution pattern than in RI-logit, Nested logit
   - Cons: too many parameters to control

Thank you for your (in)attention!